measurements on each device of course still depend upon its own connections.

Once the system with adapter has been calibrated to give corrected measurements of reflection coefficient, it can be used in the usual ways for corrected measurements of transmission coefficient.

Equation (5) has another use as an arbitrary way of dealing with redundant calibration data. Suppose the network analyzer is calibrated at first with three short circuits, to improve the plausibility of the results for highly reflecting devices [1]. If then the matched load is connected, one can pretend that an adapter is involved and apply (5). A new calibration results such that reflection magnitudes are appropriately corrected at high and low values.

Acknowledgment

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Computer-Aided Design of Waveguide Multiplexers

A. E. ATIA

***Abstract*—A procedure based on an analysis algorithm and practical rules is described for the design of waveguide multiplexers. Simple rules, which enable the designer to quickly find a near-optimum solution in a small number of iteration steps, are given. An example of a 6-channel communications multiplexer, which utilizes narrow-bandpass elliptic function waveguide filters, is also included.**

Introduction

In a multicarrier communications satellite repeater, an output multiplexer is normally needed to combine the power outputs from the traveling-wave tube amplifiers. Such a multiplexer must have the smallest possible amount of loss consistent with the required flatness in the passbands of each channel and with the selectivity required for the rejection of adjacent channels. The most suitable configuration for this application is a waveguide-manifold-type multiplexer such as that shown in Fig. 1.

Various relatively simple decoupling techniques have been previously described for the design of such multiplexers [1]. However, these techniques were found to be unsuitable for the present application, partly because extra decoupling resonators may be needed, thus increasing the size, weight, and loss of the multiplexer, and partly because the guard bands are not wide enough, although the filters have narrow bandwidths.

This paper describes a method for computer-aided design of the multiplexer. When separately and individually connected to a matched load and driven by a matched source, all filters used have the same low-pass normalized prototype characteristics. Hence, each filter may be separately tuned prior to multiplexer assembly, thus considerably reducing the effort involved in practical alignment of the multiplexer. Harmful interaction between the filters is eliminated by properly spacing them along the waveguide manifold. This approach closely simulates the process which would be followed in practical experimental design of the multiplexer.

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The author is with COMSAT Laboratories, Clarksburg, Md. 20734.

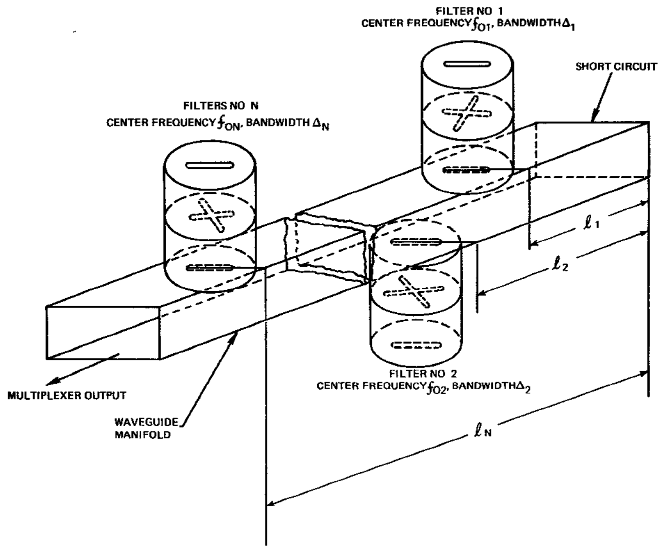

Fig. 1. Waveguide-manifold-type multiplexer.

Multiplexer Configuration and Analysis

The multiplexer to be designed consists of N filters mounted on the wide side of a rectangular waveguide manifold, as shown in Fig. 1. The waveguide is short circuited at one end, while the other end, or output port, is terminated in a matched load. All of the filters are derived from the same normalized prototype equivalent, although they have different center frequencies and bandwidths. The filters are numbered $1,2,\cdots,N$, with filter number 1 nearest to the short circuit and filter N farthest from it. The distance of the centers of the coupling slots of filter number k from the short-circuit end of the waveguide manifold is l_k; its center frequency is f_{0k} and its bandwidth is Δ_k, $k = 1,2,\cdots,N$.

The equivalent circuit of the configuration shown in Fig. 1 can be derived from an equivalent circuit of the filters, such as that shown in Fig. 2 [2], and the equivalent circuit of a T junction of the broad wall of a waveguide. Each of the T junctions may be represented by an E-plane connection [3]. The equivalent circuit parameters B_a and B_b are the same as in [3, p. 365]. Thus the complete equivalent circuit of the multiplexer is as shown in Fig. 3, in which the filters are represented by their lumped element equivalent circuit, the waveguide is represented by dispersive lengths of transmission line, and the junction effects by the susceptances B_a and B_b.

For convenience in the analysis, a set of total voltages and currents and an equivalent set of incident and reflected waves at the junctions of the filter's terminal planes and the waveguide are used in Fig. 3. Furthermore, the analysis is performed for a channel separating multiplexer rather than a summing multiplexer. All impedance levels are normalized to the waveguide characteristic impedance, which is assumed to be unity, and all filters are terminated in equal output loads R_0. Any individual filter can be analyzed to yield its Y parameters when it is considered to be a 2-port network. Only four normalized polynomials, the filter bandwidth, and the center frequency are needed to obtain the Y parameters of any filter [2]. Thus if the kth filter has terminal voltages and currents $[V_1^{(k)},V_2^{(k)}]$ and $[I_1^{(k)},I_2^{(k)}]$, respectively, then the Y matrix of the filter imposes the following constraints:

$$\begin{bmatrix} I_1^{(k)} \\ I_2^{(k)} \end{bmatrix} = \begin{bmatrix} Y_{11}^{(k)} & Y_{12}^{(k)} \\ Y_{12}^{(k)} & Y_{22}^{(k)} \end{bmatrix} \begin{bmatrix} V_1^{(k)} \\ V_2^{(k)} \end{bmatrix}$$

and, at the output terminals,

$$V_2^{(k)} = -I_2^{(k)}R_0. \tag{2}$$

Equations (1) and (2) can be solved to yield

$$I_1^{(k)} = U_k V_1^{(k)} \tag{3}$$

where

$$U_k = Y_{11}^{(k)} - \frac{Y_{12}^{(k)2}R_0}{1 + Y_{22}^{(k)}R_0}. \tag{4}$$

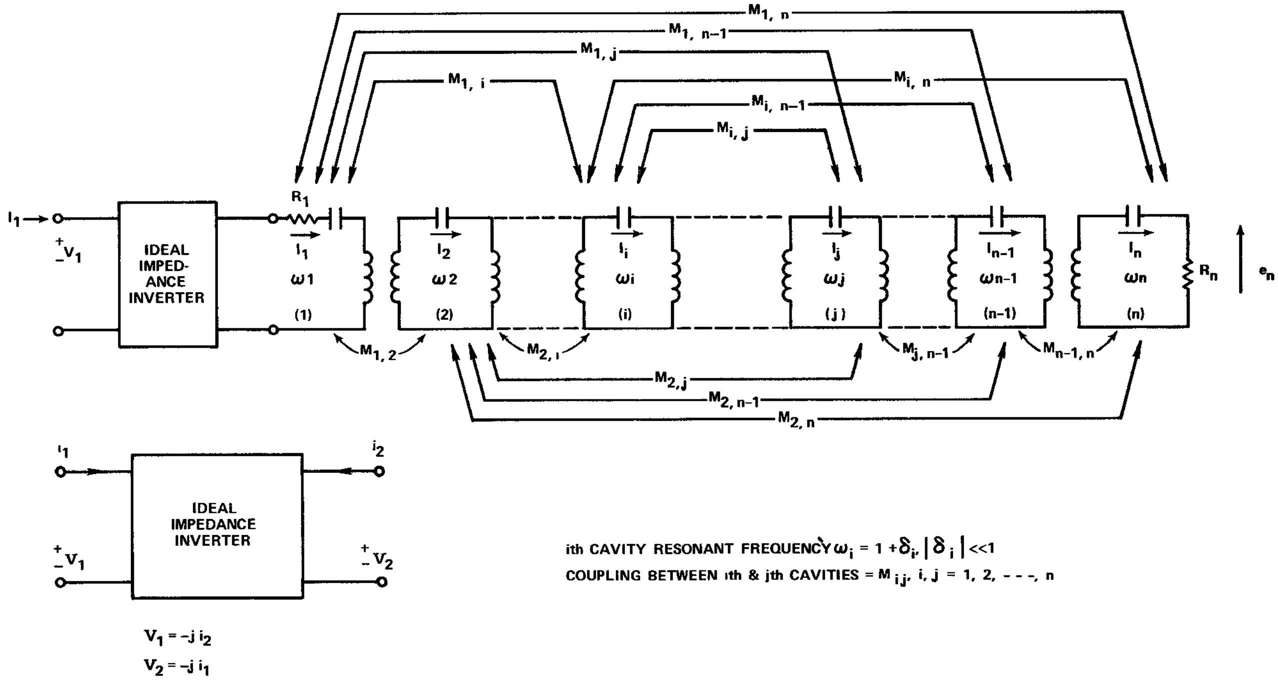

Fig. 2. Equivalent circuit of a multiple coupled cavity waveguide filter.

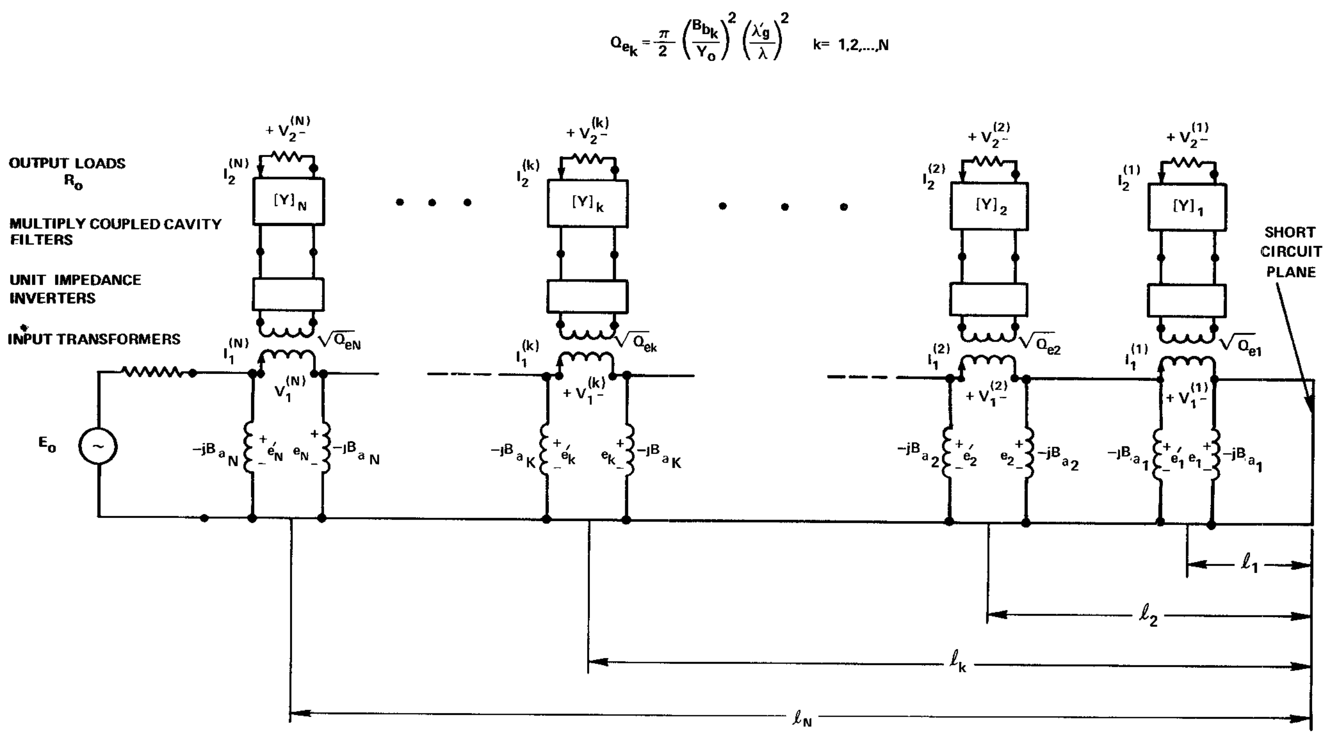

Fig. 3. Equivalent circuit of a series connected waveguide multiplexer.

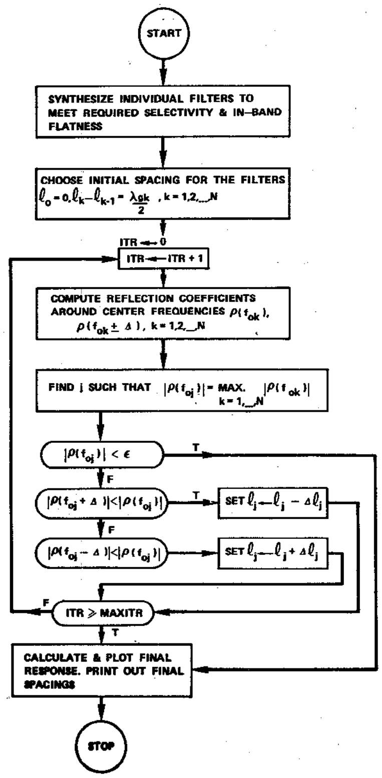

Fig. 4. Design procedure flow chart.

FILTER'S k	POSITION ℓ_k
1	2.203
2	4.319
3	6.353
4	8.314
5	10.213
6	12.051

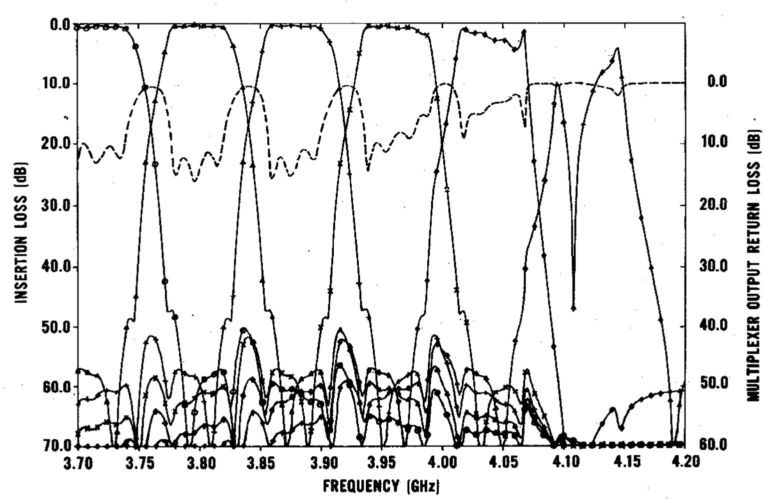

Fig. 5. Multiplexer insertion and return loss with initial spacings.

FILTER'S k	POSITION ℓ_k
1	1.830
2	4.090
3	6.145
4	8.145
5	10.045
6	12.051

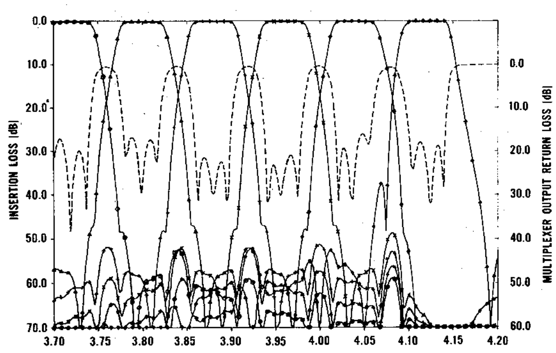

Fig. 6. Multiplexer insertion and return loss with final spacings.

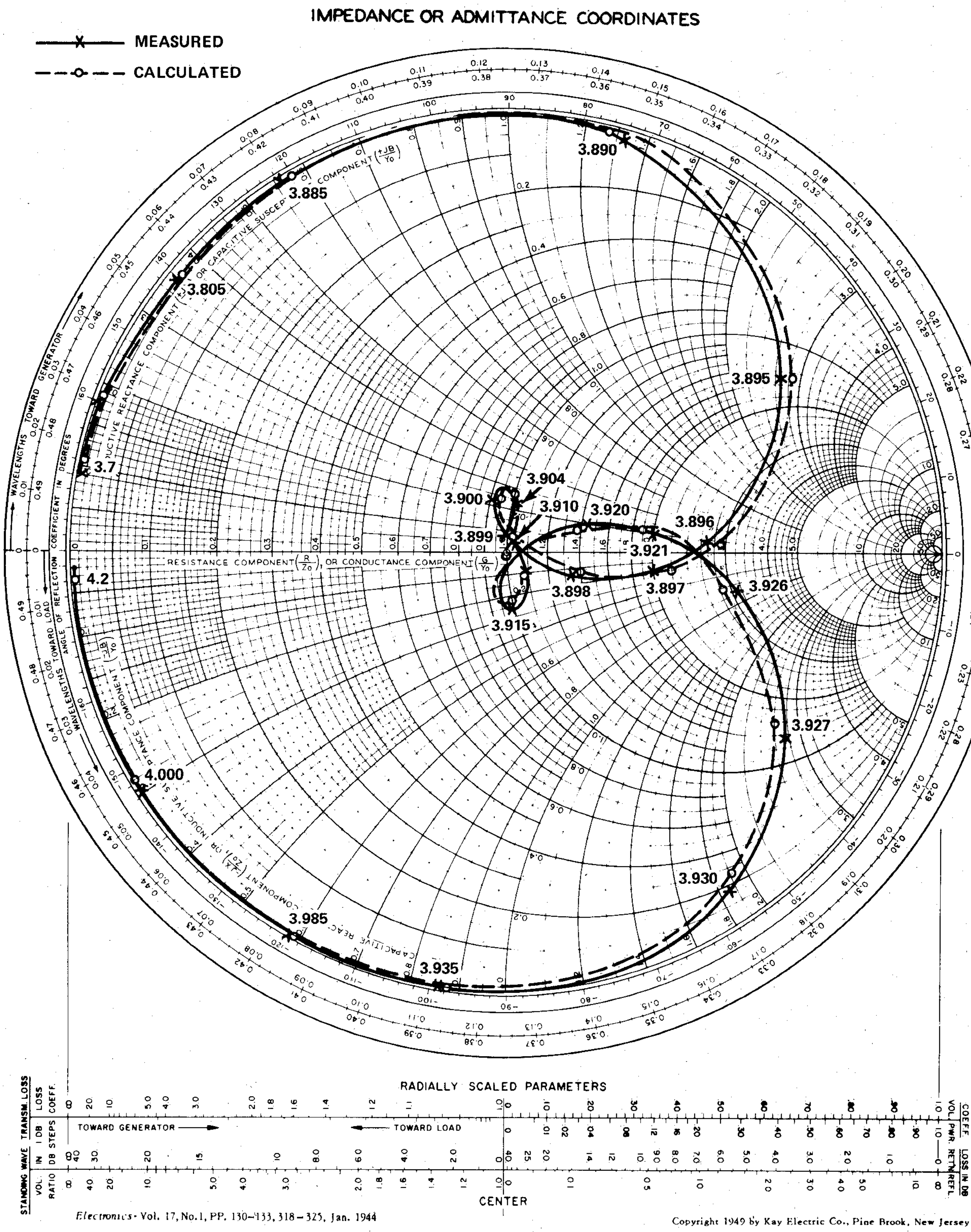

Fig. 7. Input reflection coefficient of a typical filter.

Assuming a unit incident voltage wave at the plane of the short circuit in the waveguide, the following algorithm can be used to compute the relevant quantities defined in Fig. 3:

$$V_{inc}^{(0)\prime} = 1 \quad (5a)$$

$$V_{ref}^{(0)\prime} = -1 \quad (5b)$$

$$l_0 = 0. \quad (5c)$$

Then,

$$V_{\substack{inc\\ref}}^{(k)} = V_{\substack{inc\\ref}}^{(k-1)\prime} \exp\left[\frac{\pm j2\pi(l_k - l_{k-1})}{\lambda_g}\right] \quad (6a)$$

$$\begin{matrix} e_k \\ i_k \end{matrix} = V_{inc}^{(k)} \pm V_{ref}^{(k)} \quad (6b)$$

$$I_1^{(k)} = i_k - jB_{ak}e_k \quad (6c)$$

$$V_1^{(k)} = \frac{I_1^{(k)}}{U_k} \quad (6d)$$

$$e_k' = V_1^{(k)} + e_k \quad (6e)$$

$$i_k' = I_1^{(k)} - jB_{ak}e_k' \quad (6f)$$

$$V_{\substack{inc\\ref}}^{(k)\prime} = \tfrac{1}{2}[e_k' \pm i_k'] \quad (6g)$$

where $k = 1,2,\cdots,N$, and λ_g is the guide wavelength. The generator voltage E_0, the voltage insertion loss ratio t_k of the kth filter, and the input reflection coefficient ρ are given, respectively, by

$$E_0 = I_1^{(N)} + e_N' \quad (7)$$

$$t_k = 2\left(\frac{1}{R_0}\right)^{1/2} \frac{V_2^{(k)}}{E_0} = -2(R_0)^{1/2} \frac{Y_{12}^{(k)}}{1 + R_0Y_{22}^{(k)}} \frac{V_1^{(k)}}{E_0} \quad (8)$$

$$\rho = \frac{e_N' - I_1^{(N)}}{e_N' + I_1^{(N)}}. \quad (9)$$

Design Procedure

The design procedure, summarized by the flow chart of Fig. 4, is described as follows.

1) Synthesize individual bandpass filters to meet the required selectivity and in-band flatness of the multiplexer specifications [2], [4].

2) Choose initial spacings for the filters. These spacings can be either $l_k = k\lambda_{gk}/2$, or according to the rule, $l_0 = 0$, $l_k - l_{k-1} = \lambda_{gk}/2$, $k = 1,2,\cdots,N$, where λ_{gk} is the guide wavelength at the center frequency f_{0k} of filter number k.

3) Compute the frequency response of the multiplexer using the analysis algorithm described in the previous section.

4) Find j so that

$$|\rho(f_{0j})| = \max_{k=1,2,\cdots,N} |\rho(f_{0k})|. \quad (10)$$

5) If $|\rho(f_{0j})| < \epsilon$ (a prespecified allowable reflection coefficient), then all reflection coefficients are acceptable. Print out the results and stop; otherwise, continue to step 6).

6) Change the spacing l_j of filter number j according to the following rule.

If $|\rho(f_{0j} + \Delta)| \leqslant |\rho(f_{0j})|$, then set the new value of l_j equal to $l_j + \Delta l_j$, where

$$\Delta l_j = \lambda_{gj}\left[1 - \frac{\lambda_g(f_{0j} \pm \Delta)}{\lambda_{gj}}\right] \quad (11)$$

and $\lambda_g(f_{0j} \pm \Delta)$ is the guide wavelength at frequency $(f_{0j} \pm \Delta)$.

7) If the allowable number of iterations has been exceeded, stop; otherwise, return to step 3).

The convergence of the iteration procedure to an acceptable solution was fairly rapid in all cases tested. This can be attributed to the fact that, although the initial choice of spacings does not produce the desired response, it is not very far from being optimum. The rule for changing the spacings is similar to the procedure for an empirical design approach. Namely, at each step, the filter having the worst return loss is moved by an amount which will move the position of the best return loss to its center frequency.

Example and Discussion

The above procedure has been used in the design of a 6-channel multiplexer. The filters used are 4-pole elliptic-function-type filters having 0.05-dB ripple and 42-MHz bandwidth. The insertion and return losses with the initial spacings and after the application of the optimization procedure are shown in Figs. 5 and 6, respectively. The final result of Fig. 6 was obtained after moving every filter at least twice (two iteration cycles).

The success of the procedure from a practical point of view depends largely on how closely the equivalent circuit models for the filters of Fig. 2 and the junction of [3] represent the actual behavior of these elements over the entire frequency band of the multiplexer. Fig. 7 compares the calculated and measured input reflection coefficients of a typical filter. Although a complete multiplexer assembly designed according to the procedure has not been made, the close agreement of the measured characteristics of a single filter and the computed response indicates that the present approach should yield a satisfactory practical design.

Conclusion

A procedure for the computer-aided design of waveguide multiplexers has been described. This method is based on an analysis algorithm for the equivalent circuit of the multiplexer. Simple rules for the optimization of the filter spacings allow the optimum design to be obtained in a small number of iteration steps. The filters used in the multiplexer can be either direct-coupled (Chebychev) or multiple-coupled (elliptic function) cavity filters.

An example of a 6-channel waveguide multiplexer for the frequency band of 3.7–4.2 GHz is included.

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Microwave Circuit Optimization Employing Exact Algebraic Partial Derivatives

GEORGE R. BRANNER

***Abstract*—A technique for the optimization and sensitivity analysis of broad classes of electrical networks is illustrated. The method utilizes the exact algebraic partial derivatives of functions with respect to any desired independent variable. This completely automated technique has the obvious advantage that the derivatives of any circuit response function with respect to any desired component parameter may be obtained with no additional analytical effort on the part of the designer. Several examples are given to illustrate the procedure.**

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The author is with the Department of Electrical Engineering, University of California, Davis, Calif., and ESL, Inc., Sunnyvale, Calif.